

Boxing with Konishi

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Abstract: The spin chain formulation of the operator spectrum of the $\mathcal{N} = 4$ super Yang-Mills theory is haunted by the problem of “wrapping”, i.e. the inapplicability of the formalism for short spin chain length at high loop-order. The first instance of wrapping concerns the fourth anomalous dimension of the Konishi operator. While we do not obtain this number yet, we lay out an operational scheme for its calculation. The approach passes through a five- and six-loop sector. We show that all but one of the Feynman integrals from this sector are related to five master graphs which ought to be calculable by the method of partial integration. The remaining supergraph is argued to be vanishing or finite; a numerical treatment should be possible. The number of numerator terms remains small even if a further four-loop sector is included. There is no need for infrared rearrangements.

1 Introduction

The maximally supersymmetric gauge theory in four dimensions (“ $\mathcal{N} = 4$ SYM”) has been conjectured to be dual to IIB super string theory on $\text{AdS}_5 \times \text{S}^5$ [1]. The AdS/CFT correspondence is a weak/strong coupling duality, i.e. the strong coupling limit of the gauge theory is described by the string theory. On the other hand, a direct comparison of the two sides of the duality is rarely possible because the technically accessible regimes in the two theories are mutually orthogonal.

More recently, in [2] a special limit of the original construction was discussed, in which the string theory can be consistently quantised by standard techniques. In the field theory a special set of composite operators was identified (“BMN operators”) whose planar one-loop anomalous dimension agrees with the lowest contribution in an expansion of the string energy levels in terms of an effective coupling constant. In [3] this one-loop calculation was re-interpreted in terms of an integrable spin chain model. Subsequently, the associated Bethe ansatz has been generalised to higher loop orders [4]. The inclusion of “stringy” effects is possible on the expense of invoking an interpolating factor, the “dressing phase” [5].

The spin chain interpretation is quite transparent for the BMN operators: These are products of very many elementary fields, all of which are of the same type barring for a few “impurities”. The impurities are viewed as excitations of the chain while the other fields count as empty sites. The one-loop Feynman graphs define an interaction Hamiltonian for the dynamics of the chain. This leads to a standard integrable model; higher loop diagrams yield a perturbation of its Hamiltonian. The associated Bethe ansatz can be written in closed form when the spin chain is assumed to have infinite length (“asymptotic regime”) [4]. Usually, no all-loops estimate can be extracted for operators of finite length: At high enough loop order the graphs may eventually couple to as many outer legs as there are in an operator. Beyond this order the asymptotic Hamiltonian (and with it the Bethe ansatz) ceases to be applicable (“wrapping”). Up to date, there is no systematic understanding of this situation.

The asymptotic higher-loop Bethe ansatz including the dressing phase has been tested at four loops [6]. At this order we also find the first case of wrapping: The four-loop anomalous dimension of the so-called Konishi operator cannot be obtained from the spin chain model. In order to help understanding the wrapping regime it seems vital to calculate this number. In particular, we hope to vindicate a prediction [7] based on BFKL physics [8]; it would be very much worthwhile to develop the latter approach if the outcome of the test was positive. From the point of view of the integrable model on the string side, an attempt has been made to analyse the wrapping regime by the thermodynamic Bethe ansatz [9]; also here hard data would be very much welcome.

The action of the $\mathcal{N} = 4$ SYM model with gauge group $SU(N)$ formulated in terms of $\mathcal{N} = 1$ superfields has the form

$$\begin{aligned} S_{\mathcal{N}=4} &= \int d^4x d^2\theta d^2\bar{\theta} \text{Tr} (e^{gV} \bar{\Phi}_I e^{-gV} \Phi^I) \\ &+ \left[\frac{g}{3!} \int d^4x_L d^2\theta \epsilon_{IJK} \text{Tr}(\Phi^I [\Phi^J, \Phi^K]) + c.c. \right] \\ &- \frac{1}{4g^2} \int d^4x_L d^2\theta \text{Tr}(W^\alpha W_\alpha) + \text{g.f.} + \text{ghosts}. \end{aligned} \quad (1)$$

The definition of the non-abelian field strength multiplet W_α is

$$W_\alpha = -\frac{i}{4} \bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} (e^{-gV} D_\alpha e^{gV}). \quad (2)$$

For details of our conventions we refer the reader to [10]. The vertices involving matter fields can quickly be read off from the action; for convenience we spell out the cubic YM self-interaction vertex:

$$-\frac{g}{4} \int d^4x d^2\theta d^2\bar{\theta} \text{Tr} \left([V, (D^\alpha V)] \left(-\frac{1}{4} \bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} D_\alpha V \right) \right) \quad (3)$$

In Fermi-Feynman gauge the quadratic part of the Yang-Mills action becomes $+1/2 \int V \square V$. The propagators are

$$\langle \Phi(1) \bar{\Phi}(2) \rangle = e^{i(\theta_1 \partial_1 \bar{\theta}_1 + \theta_2 \partial_1 \bar{\theta}_2 - 2\theta_1 \partial_1 \bar{\theta}_2)} \frac{1}{c_0 x_{12}^2} =: \Pi_{12}, \quad (4)$$

$$\langle V(1) V(2) \rangle = -\frac{\theta_{12}^2 \bar{\theta}_{12}^2}{c_0 x_{12}^2}. \quad (5)$$

Here 1,2 are point labels and $x_{12} = x_1 - x_2$ etc. Further, $c_0 = -4\pi^2$ so that we have the Green's function equation $\square \Pi_{12}|_{\theta_{1,2}, \bar{\theta}_{1,2}=0} = +\delta(x_{12})$.

The superspace formalism is based on the existence of two-component spinors. As a regulator we adopt supersymmetric dimensional reduction which essentially means to use $1/(x_{12}^2)^{(1-\epsilon)}$ in the propagators above in order to preserve the Green's function property. The spinor algebra remains as in four dimensions. It is not immediately obvious that this prescription is consistent at the loop-order we consider [11]; nonetheless, we proceed in good faith.

The definition of the Konishi operator is

$$\mathcal{K}_1 = \text{Tr} \left(e^{gV} \bar{\Phi}_I e^{-gV} \Phi^I \right). \quad (6)$$

On grounds of $\mathcal{N} = 1$ superconformal symmetry its superspace two-point function has the form

$$\langle \mathcal{K}_1(1) \mathcal{K}_1(2) \rangle = \frac{c(g^2)}{(\hat{x}_{L1R2}^2 \hat{x}_{L2R1}^2)^{\Delta(g^2)/2}}, \quad (7)$$

with the supersymmetry invariant combination

$$\hat{x}_{L1R2}^\mu = x_1^\mu - x_2^\mu + i((\theta_1 \sigma^\mu \bar{\theta}_1 + \theta_2 \sigma^\mu \bar{\theta}_2 - 2\theta_1 \sigma^\mu \bar{\theta}_2)). \quad (8)$$

The normalisation $c(g^2)$ and the dimension

$$\Delta(g^2) = 2 + \gamma_1 g^2 + \gamma_2 g^4 + \gamma_3 g^6 + \gamma_4 g^8 + \dots \quad (9)$$

can only be fixed by explicit perturbative calculations.

Now, we consider the ratio

$$R = \frac{(\bar{D}^2|_1 D^2|_2 \langle \mathcal{K}_1(1) \mathcal{K}_1(2) \rangle)_{\theta_{1,2}, \bar{\theta}_{1,2}=0}}{\langle \mathcal{K}_1(1) \mathcal{K}_1(2) \rangle} = -\frac{\Delta(g^2)(\Delta(g^2) - 2)}{x_{12}^2} = -\frac{2g^2\gamma_1 + \dots}{x_{12}^2}. \quad (10)$$

On the other hand, the equation of motion of the chiral field Φ^I implies

$$\bar{D}^2 \mathcal{K}_1 = -3g \mathcal{B}, \quad \mathcal{B} = \text{Tr}(\Phi^1[\Phi^2, \Phi^3]) \quad (11)$$

By tree-level perturbation theory one immediately obtains

$$\langle \mathcal{B}(1) \bar{\mathcal{B}}(2) \rangle_{\theta_{1,2}, \bar{\theta}_{1,2}=0} = -\frac{2N(N^2 - 1)}{(4\pi^2)^3 x_{12}^6}, \quad \langle \mathcal{K}_1(1) \bar{\mathcal{K}}_1(2) \rangle_{\theta_{1,2}, \bar{\theta}_{1,2}=0} = \frac{3(N^2 - 1)}{(4\pi^2)^2 x_{12}^4} \quad (12)$$

from which it follows that

$$R = -\frac{6g^2 N}{4\pi^2 x_{12}^2} \quad \Rightarrow \quad \gamma_1 = \frac{3N}{4\pi^2}. \quad (13)$$

The trick of equating the two ways of evaluating the ratio — first by differentiation of the abstract superconformal correlator, second by explicit perturbative calculation — allows one to compute the one-loop anomalous dimension of the multiplet from tree-level correlators¹ [12]. In [13] we have pushed up the method by one loop-order: We extracted two-loop anomalous dimensions γ_2 from an essentially trivial one-loop calculation. In [10] the approach was used to obtain the three-loop anomalous dimensions of the Konishi operator and a second multiplet. In the present note we discuss the suitability of the method for the calculation of the next higher value γ_4 for the Konishi multiplet.

The original work [12] did not consider a generalisation to higher loops because of a complication, the so-called Konishi anomaly [14]. The term refers to the fact that the operator \mathcal{B} mixes with

$$\mathcal{F} = \text{Tr}(W^\alpha W_\alpha). \quad (14)$$

The correct supersymmetry descendent of \mathcal{K}_1 is:

$$\mathcal{K}_{10} = \mathcal{B} + \frac{g^2 N}{32\pi^2} \mathcal{F} + \dots \quad (15)$$

¹In the original article D. Anselmi employed correlators of currents.

The difficulty lies in the fact that the classical on-shell supersymmetry transformations, if used for instance in schemes related to dimensional regularisation like in ours, do not yield the additional term when applied to \mathcal{K}_1 ; hence the term “anomaly”. It has to be fixed by independent means, i.e. by finding conformal eigenstates, or in other words by diagonalising the mixing matrix [13, 10, 15].

In [13, 10] the obstacle was circumvented by going one step higher in the multiplet: The operators \mathcal{B} and \mathcal{F} both transform into the same supersymmetry descendent

$$\mathcal{Y} = \text{Tr}([\Phi^1, \Phi^2][\Phi^1, \Phi^2]) \quad (16)$$

so that \mathcal{K}_{10} has a descendent $\mathcal{K}_{84} = a(g^2)\mathcal{Y}$. On the upper level there is no mixing, as a consequence this step is apparently anomaly-free. We switched to the ratio

$$R' = \frac{\langle \mathcal{K}_{84}(1) \bar{\mathcal{K}}_{84}(2) \rangle_{\theta_{1,2}, \bar{\theta}_{1,2}=0}}{\langle \mathcal{K}_{10}(1) \bar{\mathcal{K}}_{10}(2) \rangle_{\theta_{1,2}, \bar{\theta}_{1,2}=0}}, \quad (17)$$

again to match it with the result of differentiating an abstract superspace correlator. The superconformal argument becomes a little more involved (see [13, 10]), but the l.h.s. finally also yields $-\Delta(\Delta - 2)/x_{12}^2$.

Taylor expansion in g shows that the fourth anomalous dimension γ_4 of the Konishi operator is related to

$$d = \frac{\langle \mathcal{Y} \bar{\mathcal{Y}} \rangle_{g^6}}{\langle \mathcal{Y} \bar{\mathcal{Y}} \rangle_{g^0}} - \frac{\langle \mathcal{B} \bar{\mathcal{B}} \rangle_{g^6}}{\langle \mathcal{B} \bar{\mathcal{B}} \rangle_{g^0}}, \quad \langle \bar{\mathcal{B}} \mathcal{F} \rangle_{g^5} \quad (18)$$

and a number of lower order pieces which contain (with the exception of three further four-loop superdiagrams [10, 17]) only three-loop (or lower) Feynman-diagrams which we may disregard for now because any three-loop two-point function can be evaluated with the Mincer programme [16].

The difference d contains $O(g^6)$ diagrams, which need to be evaluated up to the finite part. The order-reduction effect is present: The $O(g^8)$ number γ_4 can be extracted from correlators of lower order. This impression is deceiving, though: In momentum space the Feynman graphs contributing to d have up to six loops, while a more direct approach, e.g. the insertion of \mathcal{K}_1 into a three-point correlator would only lead to four-loop graphs.

The point of this note is to show that the excess loop-orders of our approach are not a severe handicap, while the total number of numerator terms in the four- and higher-loop sector is $O(100)$. In comparison, the operator insertion approach requires an infrared rearrangement (which cannot easily be automated) for a number of terms that is presumably larger by more than two orders of magnitude.

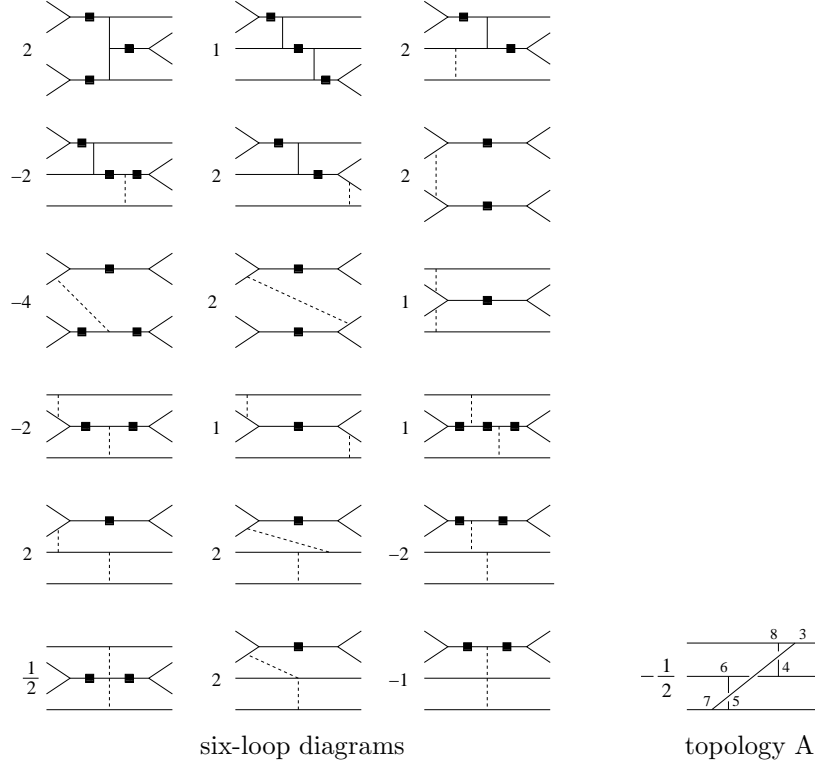
2 The set of high-loop supergraphs

We give a list of the $\mathcal{N} = 1$ supergraphs contributing to the leading order in N of the difference $d = \langle \mathcal{Y} \bar{\mathcal{Y}} \rangle_{g^6} / \langle \mathcal{Y} \bar{\mathcal{Y}} \rangle_{g^0} - \langle \mathcal{B} \bar{\mathcal{B}} \rangle_{g^6} / \langle \mathcal{B} \bar{\mathcal{B}} \rangle_{g^0}$ up to the finite part. The normalisations of the vertices and the additional minus sign on the Yang-Mills propagators have been taken into account, a global factor $12(g^2 N)^3$ has been taken out. In the diagrams below a solid line means a matter propagator as defined in (4) and a dashed line denotes a Yang-Mills propagator as defined in (5), although with the sign aligned. All external lines meet at point 1 on the left, or at point 2 on the right. We have opened up the diagrams only for convenience of drawing. Last, free lines (in numerator or denominator) have not been drawn.

The BPS operator $\text{Tr}(\Phi^1 \Phi^1 \Phi^1 \Phi^1)$ has an $SU(4)$ descendent $P = \text{Tr}(\Phi^1 \Phi^2 \Phi^1 \Phi^2) + 2 \text{Tr}(\Phi^1 \Phi^1 \Phi^2 \Phi^2)$. The N expansion of its $O(g^6)$ two-point function contains three independent linear combinations of graphs. Protectedness (the absence of quantum corrections to the canonical dimension of the operator) implies that each of these sums vanishes up to contact terms. The leading N linear combination may be used to subtract the complete pure Yang-Mills sector out of the difference d .

One-loop bubbles may be put to zero in the manifestly supersymmetric Fermi-Feynman gauge. Further, we start by checking the combinatorics of graphs involving cubic vertices only. Higher vertices lead to derived topologies whose combinatorics follows the same pattern. Ghosts drop: Since there are only matter fields at the outer points they could occur only in the loop corrections to the cubic Yang-Mills vertex or in the two-loop Yang-Mills propagator. Similar diagrams with matter loops are absent from our set of graphs, though, while the relative coefficients are fixed. At leading N this argument also rules out graphs with three or more cubic Yang-Mills vertices and derived diagrams. We have checked the combinatorics for graphs with one cubic Yang-Mills self-interaction; this sector yields two diagrams relating to

the two-loop matter propagator and the one-loop renormalised cubic matter/Yang-Mills vertex. Apart from the aforementioned propagator or vertex corrections one could indeed draw a planar four-loop graph involving two cubic Yang-Mills self-interaction vertices (an H shape between two matter lines). But the interaction does not connect to three or four matter lines so that this diagram will certainly be eliminated upon subtracting the protected linear combination; a similar matter graph does in fact drop out.



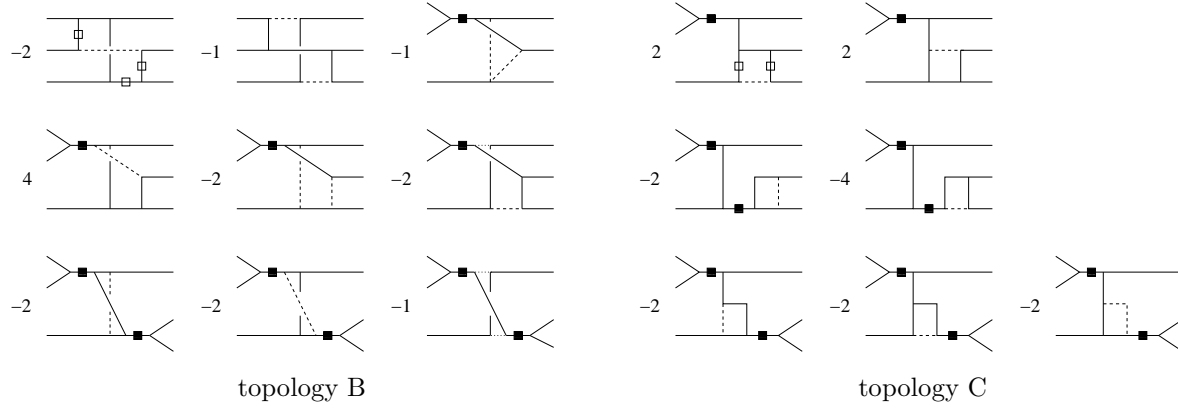
The six-loops diagrams all have numerators of the type $\square (\partial + \partial)^2 (\partial + \partial)^2$ or simpler (in the round brackets we mean two different partial derivatives acting on non-adjacent lines); in most cases one or even both of the round brackets are also replaced by box operators. In the figure above we have marked all the d'Alembertians by black squares on the lines which they remove. It is easy to see that the resulting derived topologies could equivalently be obtained from the two master graphs



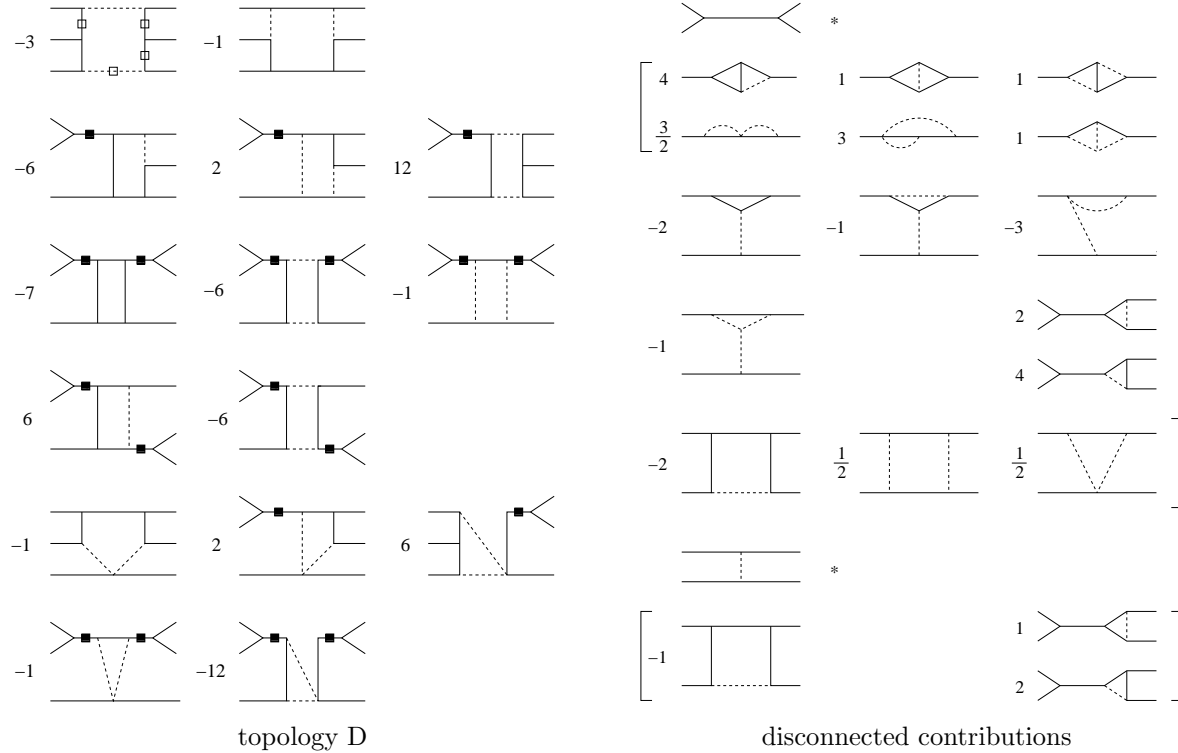
by placing the same number of box operators as before, i.e. the total number of derivatives remains six. Both six-loop masters should be calculable by the partial integration technique [16]: They have triangle subgraphs and the reduction should lead to structures known in the literature. Where there is a choice, the second master is preferable because it has three triangles. The six-loop supergraphs in the middle column are related to the first master; only the bottom one can also be derived from the second master. All other six-loop supergraphs can be put into the second category.

The five-loop sector counts 32 superdiagrams, some of which come with rather complicated numerators. Nevertheless, one can check without calculation that the graphs fall into classes related to only four master topologies: We are interested in the $\theta_{1,2} = 0 = \bar{\theta}_{1,2}$ component of the supergraphs. The Grassmann integration at a matter vertex with two outer legs will then produce a box operator on the third leg. Below we have again indicated these d'Alembertians by a black square on the corresponding line. On the master graphs (the first diagram of each category) we have marked by a white square which

lines should be shrunk to obtain the set of derived graphs.



Despite of the fact that the graphs of topology A and B are non-planar, their group factor is of leading order in N in the correlator $\langle \mathcal{B}\bar{\mathcal{B}} \rangle_{g^6}$. The colour structure of the operator B is an f^{abc} symbol. The non-planar graphs can all be drawn in the plane when one line is pulled around an outer operator; put in a different way, if the operator is drawn inside the graphs like a cubic vertex. These diagrams are suppressed by $1/N^2$ in the $\langle \mathcal{Y}\bar{\mathcal{Y}} \rangle_{g^6}$ two-point function, so that they do not drop by forming the difference of ratios. The occurrence of the non-planar sector at leading order in N is the essence of “wrapping” and it is at this loop-order unique to the Konishi multiplet.



The last figure shows a number of disconnected graphs that also occur in the difference of correlation functions d . They present no computational problem since the maximum loop-order is three. The master diagrams of the B,C, and D topologies all have triangle subgraphs and thus they can hopefully be evaluated by the method of partial integration [16].

3 Finiteness of topology A

From this point of view the single diagram of topology A presents a major complication as it does not contain a single triangle.

In the list of diagrams above we have used a protected linear combination to suppress the pure Yang-Mills sector. Anomalous dimension is caused by graphs with some colour (or equivalently flavour) antisymmetrisation; this is clearly visible in all the graphs in topology B,C and D that contain a matter vertex with two outer legs, and thus an f^{abc} symbol contracted on the group factor of the outer operator. None of these diagrams occurs in protected linear combinations. Closer inspection of the colour factors shows that the same is true for the master graphs of type D.

By taking an appropriate difference of the leading N contributions of the $O(g^6)$ two-point functions of the half BPS operators $\text{Tr}(\Phi^1\Phi^1\Phi^1\Phi^1)$ (used above) and $\text{Tr}(\Phi^1\Phi^1\Phi^1)$ one can form a protected linear combination of very few integrals that contains some disconnected diagrams, the two six-loop master integrals, the very matter diagram of topology A listed above, and a pure YM version thereof and of the non-planar B master. It is possible to use this to eliminate the matter A graph. However, the pure Yang-Mills A graph has an equally complicated numerator, and in addition we would switch on other parts of the pure Yang-Mills sector. Further protected combinations involving the matter A diagram occur at subleading orders in N e.g. in the $\text{Tr}(\Phi^1\Phi^1\Phi^1\Phi^1)$ two-point function, but these would bring in other perhaps even more complicated non-planar diagrams.

The very fact that the matter A graph does occur in protected linear combinations proves useful on its own: We have observed on many occasions (c.f. [18], albeit in an $\mathcal{N} = 2$ context) that superdiagrams in two-point functions of protected operators tend to be individually finite or even vanishing if they have sufficiently many outer legs. In this section we argue that the same should apply to the matter sector A graph. We would then hope that the leading contribution can be found numerically with good precision — e.g. with the help of the package [19] — and, in particular, that it should be clearly visible if the graph vanished.

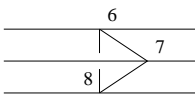
Expanding the exponential shift operators of the matter superpropagators is very straightforward since all integration points have definite chirality. We find the numerator

$$\text{num}(\text{topo}_A) = -\square_{37}\square_{48}\square_{56} - \square_{38}\square_{46}\square_{57} - \text{Tr}(\partial_{37}\partial_{38}\partial_{48}\partial_{46}\partial_{56}\partial_{57}) \quad (19)$$

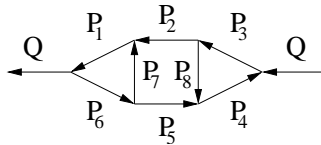
where ∂_{ij} denotes a partial derivative acting at point i on a propagator between points i, j . In the last formula we have only given the derivatives which act under the integrals on the integrand

$$\text{topo}_A = \int \frac{d^4 x_{3,4,5,6,7,8}}{c_0^{12} x_{16}^2 x_{17}^2 x_{18}^2 x_{23}^2 x_{24}^2 x_{25}^2 x_{37}^2 x_{38}^2 x_{46}^2 x_{48}^2 x_{56}^2 x_{57}^2} \quad (20)$$

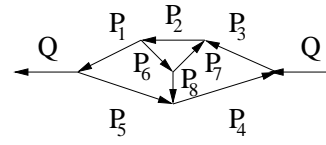
For the rest of this note we give integrands pictorially while numerators are written in derivative form.



the football graph



the ladder topology



the Benz topology

Next, both triple box terms in the numerator (19) reduce the six-fold integration in (20) to

$$\text{football} = \int \frac{d^4 x_{6,7,8}}{c_0^9 x_{16}^2 x_{17}^2 x_{18}^2 x_{26}^2 x_{27}^2 x_{28}^2 x_{67}^2 x_{68}^2 x_{78}^2}. \quad (21)$$

We note that the subintegral involving points 7,8 is finite and conformal:

$$\int \frac{d^4 x_{7,8}}{x_{17}^2 x_{27}^2 x_{67}^2 x_{78}^2 x_{18}^2 x_{28}^2 x_{68}^2} \propto \frac{\zeta(3)}{x_{12}^2 x_{16}^2 x_{26}^2} \quad (22)$$

so that the remaining integration in the “football” graph leads to the divergent integral

$$g(1, 1, 2, 2) = \int \frac{d^4 x_6}{x_{16}^4 x_{26}^4}. \quad (23)$$

In x -space dimensional regularisation this causes a simple pole. The conformal symmetry of the $x_{7,8}$ -subintegral is broken by the regulator, but this does not affect our conclusion about the leading term, which must be proportional to $\zeta(3)/(\epsilon x_{12}^6)$.

The trace of six Pauli matrices contains contributions involving three η -symbols (the flat metric) and a second type of terms with one η and one four-index totally antisymmetric tensor. The latter terms must eventually cancel because they would lead to a pseudoscalar contribution, while the integral can finally only depend on the single difference variable x_{12} .

The relevant terms of the six-trace may be grouped as follows:

$$\begin{aligned} \frac{1}{2} \text{Tr}(\sigma_{\mu_1} \sigma_{\mu_2} \sigma_{\mu_3} \sigma_{\mu_4} \sigma_{\mu_5} \sigma_{\mu_6}) = & \eta_{\mu_1 \mu_2} \eta_{\mu_3 \mu_4} \eta_{\mu_5 \mu_6} \\ & + \eta_{\mu_1 \mu_2} (\eta_{\mu_3 \mu_6} \eta_{\mu_4 \mu_5} - \eta_{\mu_3 \mu_5} \eta_{\mu_4 \mu_6}) \\ & + \eta_{\mu_3 \mu_4} (\eta_{\mu_1 \mu_6} \eta_{\mu_2 \mu_5} - \eta_{\mu_1 \mu_5} \eta_{\mu_2 \mu_6}) \\ & + \eta_{\mu_5 \mu_6} (\eta_{\mu_1 \mu_4} \eta_{\mu_2 \mu_3} - \eta_{\mu_1 \mu_3} \eta_{\mu_2 \mu_4}) \\ & + (\eta_{\mu_2 \mu_3} \eta_{\mu_4 \mu_5} \eta_{\mu_1 \mu_6} \pm 7 \text{ terms}) \end{aligned} \quad (24)$$

Here the seven omitted terms are obtained from the first one in the last line by antisymmetrising $1 \leftrightarrow 2$, $3 \leftrightarrow 4$, $5 \leftrightarrow 6$ (all terms have coefficient ± 1). As before, let us first pretend that the integral is finite. We separately consider the $x_{3,4,5}$ subintegrals. Each of them is differentiated on two legs, where the derivatives are either contracted or antisymmetrised. In the first case we may use partial integration and the box operation to break the integral, in the second case we employ

$$(\partial_{x_7^\mu} \partial_{x_8^\nu} - \partial_{x_8^\mu} \partial_{x_7^\nu}) \int \frac{d^4 x_3}{x_{37}^2 x_{38}^2 x_{23}^2} = -c_0 \frac{x_{27}^\mu x_{28}^\nu - x_{28}^\mu x_{27}^\nu}{x_{27}^2 x_{28}^2 x_{78}^2}. \quad (25)$$

Simply by completing the squares in the numerator we find that the six-trace reduces to twice the “football” graph, where the sign is opposite to the other terms. Hence the matter sector A diagram is predicted to vanish.

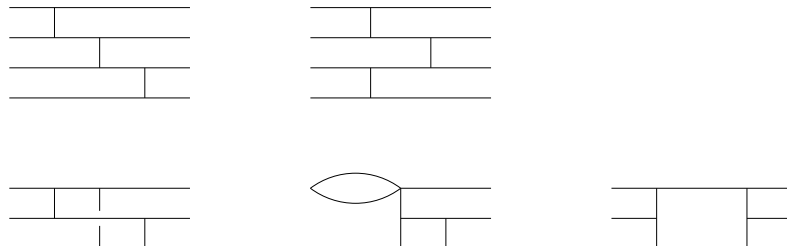
However, the argument is not sound since in the process of completing the squares we produce divergent terms, although they finally cancel. However, the leading overall singularity should come out correctly in any regularisation scheme. Now, in x -space dimensional regularisation, the box operation still removes an integral, but equation (25) is true only to leading order² in ϵ . By the symmetries of the integrand, the second, third, and fourth line of (24) actually give identical contributions. Let us repeat the exercise of completing the squares separately for the terms coming from the second line and those from the fifth. The triple box terms from the first line form a third group of terms. It turns out that the only structure with nine propagator factors is the football graph, in all other terms in the three groups one propagator factor is cancelled by the respective numerator. When one of the interior lines is missing, say $1/x_{78}^2$, we put $q = x_{12}, p_1 = x_{27}, p_2 = x_{26}, p_3 = x_{28}$ to find a three-loop ladder topology, else upon a similar identification we obtain a Benz-graph. We may evaluate these by the Mincer programme to get an idea about the leading singularities. Subleading terms cannot be expected to be correct, because the Mincer algorithm calculates with integer powers of denominator factors, while x -space dimensional regularisation introduces fractional powers. In the triple-box and the single-box groups we find the football, and ladder and Benz contributions. We put the football aside, of which we know that the leading singularity is a simple pole. In both groups of terms a double pole cancels between the ladder and the Benz parts. We believe that the remaining simple poles are significant, because the Mincer system is self-consistent so that the choice of topology does not matter. The no-box terms from the fifth line of (24) yield only ladder contributions. The sum of all terms of this group has a simple pole, too.

Subleading corrections caused by the breakdown of equation (25) would hence affect the finite part of the graph, possibly shifting it away from zero. However, it is quite suggestive that also the Mincer results exactly cancel when all three groups of terms are added up. In conclusion, we have produced evidence that the matter A graph should be finite or ideally even vanishing. For our calculation we need the ϵ -expansion of the integrals up to the finite part in x -space, so in this case only the leading order. It will hopefully be possible to obtain this value at least by numerical methods.

²We thank E. Sokatchev and Y. Stanev for discussions about this point.

4 Conclusions

The technique suggested in [12] and generalised to multi-loop level in [13, 10] yields a feasible scheme for the calculation of the four-loop anomalous dimension of the Konishi multiplet. To this end the set of five- and six-loop master-integrals



will have to be evaluated with rather general six-derivative numerators (four derivatives for topology C). We believe that this is possible using the method of partial integration. Appendix A contains the sum of the numerators of the supergraphs obtained from the Grassmann expansion. We list the derived topologies resulting from the action of box operators. Irreducible dot products are left as numerators. Clearly, the number of terms is very small in comparison to more direct calculations e.g. by insertion of the Konishi operator into a three-point function. In Appendix B we have collected all the five-loop pieces into long numerators for the B,C,D master topologies in order to better illustrate this point.

Beyond the three-loop level the completion of our project also requires the evaluation of the four-loop correlator $\langle \bar{\mathcal{B}}\mathcal{F} \rangle_{g^5}$. Choosing the chiral $SU(4)$ representatives (11), (14) for \mathcal{B} , \mathcal{F} minimizes the number of graphs. We counted sixty $\mathcal{N} = 1$ superdiagrams, thirteen of which factor into lower order pieces because they contain quartic vertices. To evaluate the numerators we found it most convenient to treat the F operator and the non-abelian self-interactions by commuting and partially integrating spinor derivatives (“ D -algebra”), but then to obtain the Grassmann expansion of these non-abelian pieces and to insert it into the rest of the graphs consisting of matter lines dealt with by the exponential shift technique. The d’Alembertians stand such that all graphs are derived topologies w.r.t. the following set of four-loop master integrals:



The latter integrals arise from the BU three-loop topology by inserting an additional line. The one-loop corrections to the central vertex also occur, but here the numerators fetch a box operator so that the graphs can in fact be sorted into the other four classes. We are currently summing up the derived topologies. From scratch, the number of terms is of similar order to the five- and six-loop sector. The masters have less independent momenta and hence less non-trivial dot products, so that simplifications may be expected to be more far-reaching.

In conclusion, our approach obviously has the inconvenience of introducing five high-loop master graphs, which a direct calculation would not contain. On the positive side, this method has no problem with spurious IR singularities, since no momentum is ever put to zero. Further, we only find a total of nine masters beyond three-loop level, and the total number of terms in their numerators is $O(100)$. A numerical evaluation [19] of the whole set of graphs could therefore give relatively good precision. If the matter A graph can reliably be checked to vanish by numerical means and the exact evaluation of the remaining graphs is possible by the partial integration method we will obtain an analytic result.

Two hidden assumptions are the consistency of dimensional reduction at this loop-order [11] and the absence of a generalised Konishi anomaly in the supersymmetry transformations leading from the \mathcal{K}_{10} operator to \mathcal{K}_{84} , c.f. [13, 10].

Acknowledgements

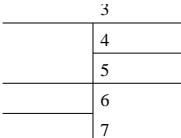
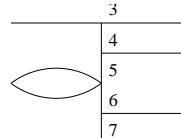
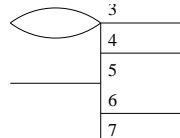
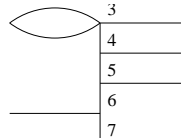
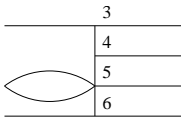
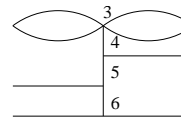
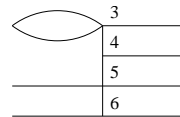
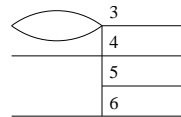
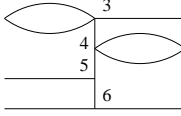
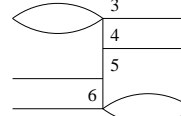
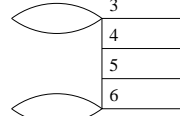
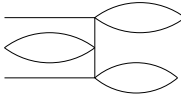
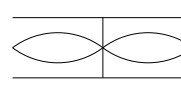
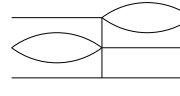
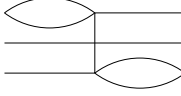
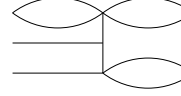
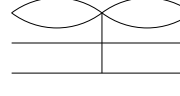
The author is grateful to S. Moch, E. Sokatchev, and V. Velizhanin for helpful discussions.

Appendix A: From supergraphs to ordinary Feynman-integrals

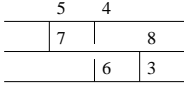
The evaluation of the numerators of the superdiagrams is simplified by partial integration. As a basis we have normally chosen the derivatives on the internal lines. In a last step, dot products of adjacent derivatives at a three-vertex may be replaced by box operators. Occasionally, we meet third or fourth derivatives on one line. In these cases we have left one box operator and partially integrated away the other derivatives. The list below was obtained by shrinking the lines hit by box operators and summing up all numerator contributions for each derived topology. Some derived diagrams are common to the B,C,D groups and/or the six-loop sector. We have exploited the symmetries of each derived topology to reduce its numerator.

The final result is unique only up to partial integration; we could not identify an ideal choice. In the pictures below every line denotes $1/(c_0 x_{ij}^2)$ and every vertex implies an integral. Points were labeled where necessary to give a meaning to the numerators.

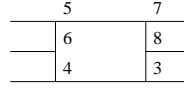
4.1 Six-loop structures

			
$-4(\partial_{16}\partial_{17})(\partial_{23}\partial_{24})$	$-4(\partial_{23}\partial_{24})(\partial_{26}\partial_{27})$	$8(\partial_{24}\partial_{56})(\partial_{26}\partial_{27})$	$-8(\partial_{16}\partial_{17})(\partial_{24}\partial_{25})$
			
$4(\partial_{23}\partial_{24})$	$4(\partial_{26}\partial_{45})$	$4(\partial_{24}\partial_{25}) + 4(\partial_{24}\partial_{26})$	$-4(\partial_{25}\partial_{26})$
			
$-4(\partial_{15}\partial_{16})$	$-4(\partial_{15}\partial_{34})$	$-4(\partial_{27}\partial_{28})$	
			
-2	$-\frac{1}{2}$	2	
			
-1	2	1	

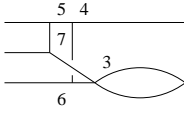
4.2 Five-loop structures



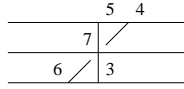
$$4(\partial_{36}\partial_{57})(\partial_{38}\partial_{46})(\partial_{45}\partial_{78}) \\ -2(\partial_{36}\partial_{57})(\partial_{38}\partial_{45})(\partial_{46}\partial_{78}) \\ +2(\partial_{36}\partial_{45})(\partial_{38}\partial_{57})(\partial_{46}\partial_{78})$$



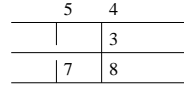
$$-12(\partial_{34}\partial_{78})(\partial_{38}\partial_{56})(\partial_{46}\partial_{57}) \\ +6(\partial_{34}\partial_{57})(\partial_{38}\partial_{56})(\partial_{46}\partial_{78}) \\ -6(\partial_{34}\partial_{57})(\partial_{38}\partial_{46})(\partial_{56}\partial_{78})$$



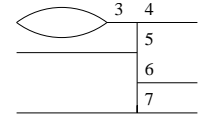
$$8(\partial_{36}\partial_{45})(\partial_{37}\partial_{45}) \\ -2(\partial_{37}\partial_{45})(\partial_{46}\partial_{57}) \\ +2(\partial_{36}\partial_{45})(\partial_{46}\partial_{57})$$



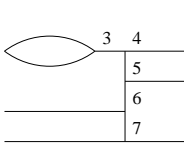
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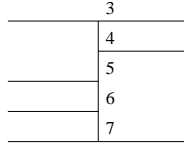
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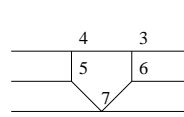
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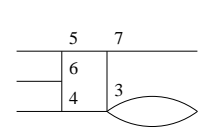
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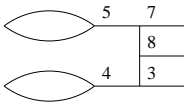
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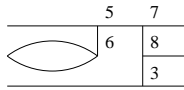
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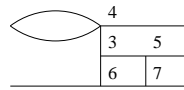
$$-12(\partial_{34}\partial_{56})(\partial_{46}\partial_{57}) \\ +12(\partial_{37}\partial_{56})(\partial_{46}\partial_{57})$$



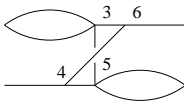
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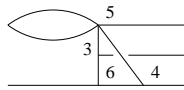
$$-12(\partial_{38}\partial_{57})(\partial_{56}\partial_{78})$$



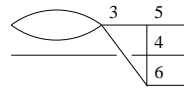
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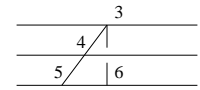
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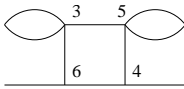
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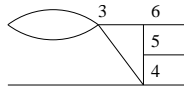
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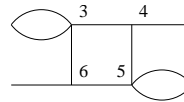
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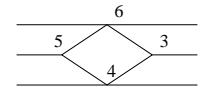
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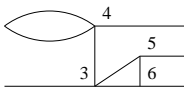
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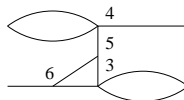
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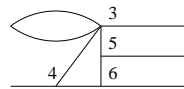
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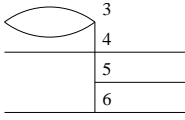
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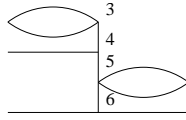
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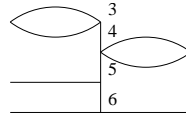
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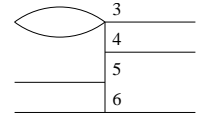
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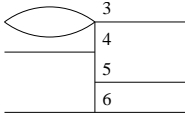
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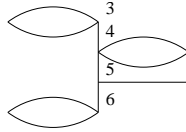
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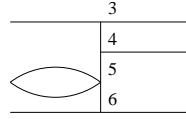
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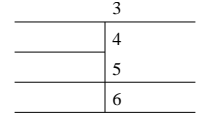
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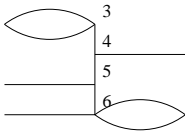
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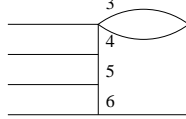
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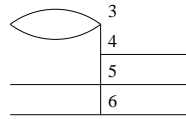
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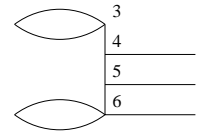
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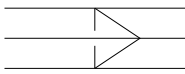
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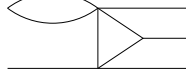
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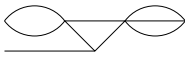
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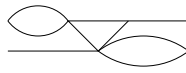
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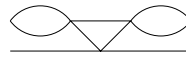
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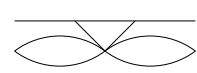
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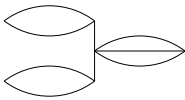
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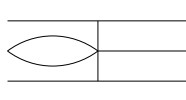
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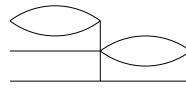
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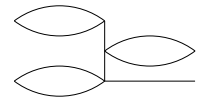
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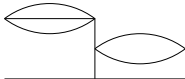
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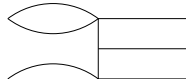
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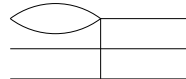
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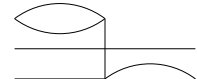
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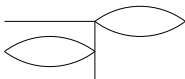
$$1$$



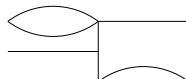
$$-\frac{7}{2}$$



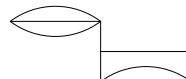
$$\frac{17}{2}$$



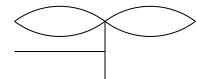
$$\frac{7}{2}$$



$$-1$$

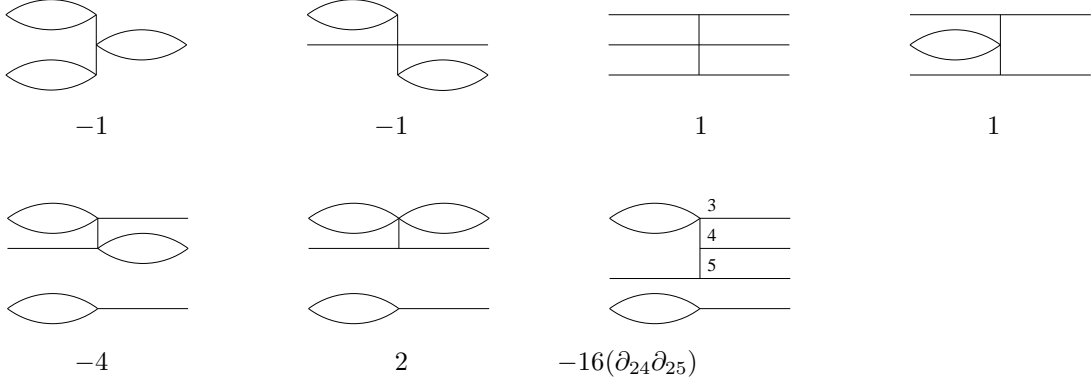


$$3$$



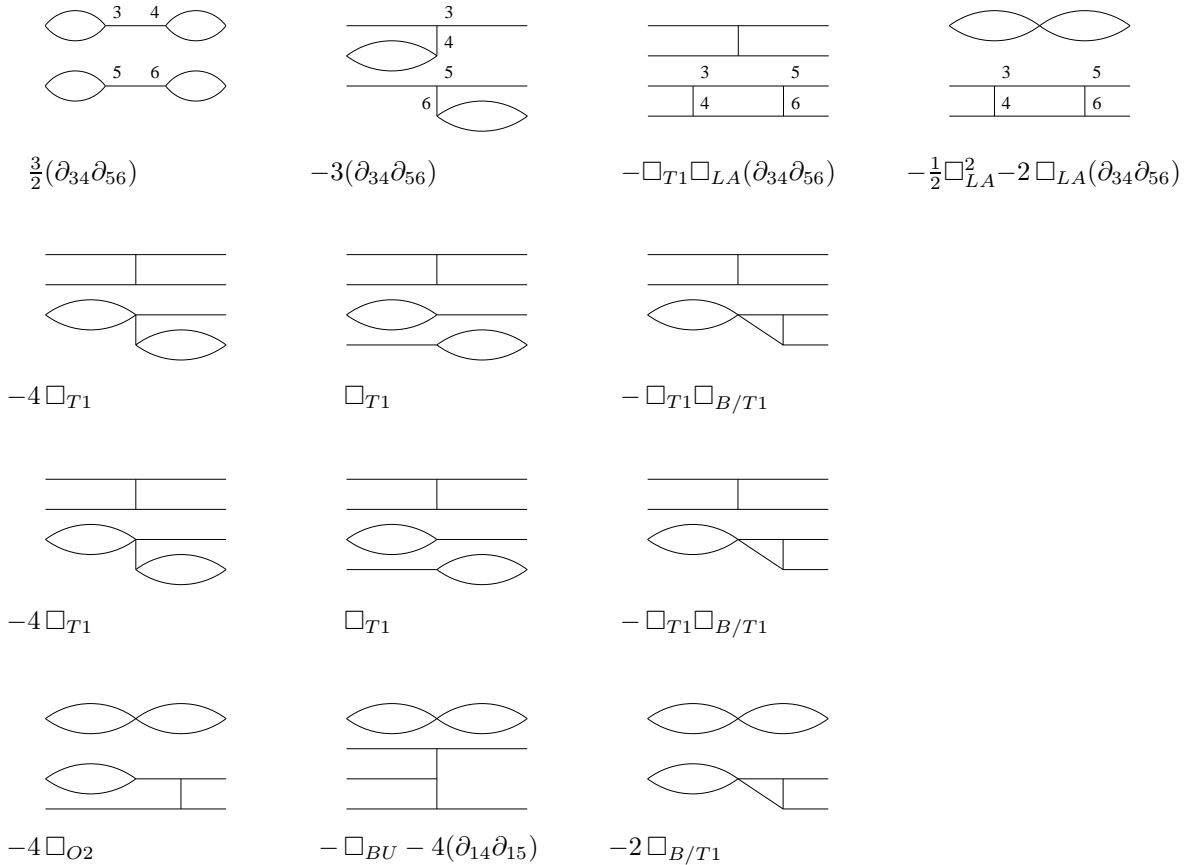
$$2$$

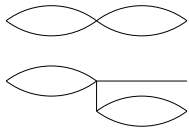
4.3 Four-loop structures



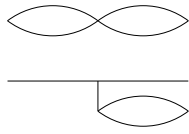
4.4 Further disconnected structures: Three loops and lower

In most of these cases it is very simple to express dot products by box-operators, where we use the square of the in-going momentum, too. To indicate this we use a box-operator indexed by the topology of the subgraph on which it acts. By $B/T1$ we mean the factorised subdiagram built from a bubble followed by a T1.

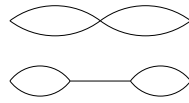




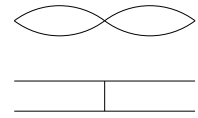
-8



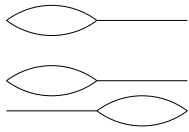
-2



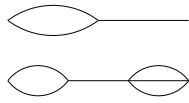
-2



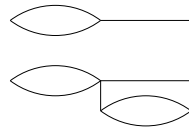
1



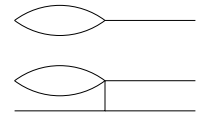
$-\frac{1}{2}$



-3



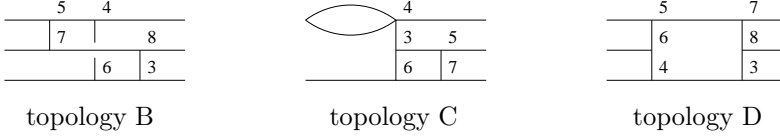
-6



4

Appendix B: Collective numerators for the five-loop part

In this section we have collected the sum of five-loop graphs from Appendix A into numerators with box operators acting on the integrands of the B,C,D master integrals. Where a choice was possible we have preferred the D master. For convenience, we repeat the point labels:



In the numerator of the B graph we have separated the non-planar part from terms with box operators yielding planar derived topologies.

$$\begin{aligned}
 \text{num}(\text{topo}_B) = & 4(\partial_{36}\partial_{57})(\partial_{38}\partial_{46})(\partial_{45}\partial_{78}) - 2(\partial_{36}\partial_{57})(\partial_{38}\partial_{45})(\partial_{46}\partial_{78}) + 2(\partial_{36}\partial_{45})(\partial_{38}\partial_{57})(\partial_{46}\partial_{78}) \\
 & + \square_{38} (8(\partial_{36}\partial_{45})(\partial_{37}\partial_{45}) - 2(\partial_{37}\partial_{45})(\partial_{46}\partial_{57}) + 2(\partial_{36}\partial_{45})(\partial_{46}\partial_{57})) \\
 & + \square_{78} (-4(\partial_{37}\partial_{46})(\partial_{46}\partial_{57}) - 2(\partial_{36}\partial_{45})(\partial_{37}\partial_{46})) + \square_{36} (-4(\partial_{34}\partial_{57})(\partial_{45}\partial_{78})) \\
 & + \square_{38}\square_{57} (-(\partial_{46}\partial_{78})/2 + 4(\partial_{36}\partial_{45})) + \square_{57}\square_{78} (-4(\partial_{38}\partial_{46}) + 4(\partial_{36}\partial_{45})) \\
 & + \square_{38}\square_{45} (2(\partial_{36}\partial_{78}) + 2(\partial_{36}\partial_{57})) + \square_{36}\square_{45} (-3(\partial_{46}\partial_{78})/2) \\
 & + \square_{36}\square_{45}\square_{78}(-1/2) \\
 & - - - \\
 & + \square_{17} (2(\partial_{36}\partial_{45})(\partial_{38}\partial_{46})) + \square_{15} (2(\partial_{36}\partial_{78})(\partial_{38}\partial_{46})) \\
 & + \square_{16} (-2(\partial_{38}\partial_{57})(\partial_{45}\partial_{78})) \\
 & + \square_{15}\square_{38} (4(\partial_{16}\partial_{45}) + (\partial_{46}\partial_{78})) + \square_{15}\square_{78} (-4(\partial_{46}\partial_{78})) \\
 & + \square_{16}\square_{57} (-4(\partial_{36}\partial_{78}) + (\partial_{38}\partial_{45})) + \square_{16}\square_{78} (-4(\partial_{46}\partial_{57}) + (\partial_{38}\partial_{45})) \\
 & + \square_{17}\square_{38} (4(\partial_{16}\partial_{45})) + \square_{16}\square_{38} (-4(\partial_{45}\partial_{78})) + \square_{15}\square_{36} (2(\partial_{46}\partial_{78})) \\
 & + \square_{16}\square_{57}\square_{78}(-1/2) + \square_{17}\square_{36}\square_{46}(4) + \square_{15}\square_{36}\square_{78}(-7/2) \\
 & + \square_{16}\square_{38}\square_{57}(7/2) + \square_{17}\square_{38}\square_{45}(-1) + \square_{15}\square_{38}\square_{78}(2)
 \end{aligned} \tag{26}$$

$$\begin{aligned}
 \text{num}(\text{topo}_C) = & -8(\partial_{16}\partial_{34})(\partial_{36}\partial_{57}) \\
 & + \square_{36} (4(\partial_{25}\partial_{27})) + \square_{57} (4(\partial_{35}\partial_{67})) + \square_{16} (-4(\partial_{34}\partial_{57})) \\
 & + \square_{57}\square_{67}(2)
 \end{aligned} \tag{27}$$

$$\begin{aligned}
 \text{num}(\text{topo}_D) = & -12(\partial_{34}\partial_{78})(\partial_{38}\partial_{56})(\partial_{46}\partial_{57}) + 6(\partial_{34}\partial_{57})(\partial_{38}\partial_{56})(\partial_{46}\partial_{78}) - 6(\partial_{34}\partial_{57})(\partial_{38}\partial_{46})(\partial_{56}\partial_{78}) \\
 & + \square_{57} (-4(\partial_{34}\partial_{56})(\partial_{34}\partial_{78}) + 10(\partial_{34}\partial_{56})(\partial_{38}\partial_{46})) \\
 & + \square_{38} (-12(\partial_{34}\partial_{56})(\partial_{46}\partial_{57}) - 12(\partial_{46}\partial_{57})(\partial_{56}\partial_{78})) \\
 & + \square_{16} (6(\partial_{34}\partial_{78})(\partial_{38}\partial_{57})) + \square_{14} (-12(\partial_{38}\partial_{57})(\partial_{56}\partial_{78})) \\
 & + \square_{56}\square_{78} (-3(\partial_{34}\partial_{57})) + \square_{34}\square_{56} (-5(\partial_{46}\partial_{78}) + 8(\partial_{38}\partial_{57})) \\
 & + \square_{38}\square_{56} (-3(\partial_{34}\partial_{57}) + 3(\partial_{37}\partial_{45})) + \square_{34}\square_{57} (2(\partial_{38}\partial_{46}) - 2(\partial_{38}\partial_{56})) \\
 & + \square_{56}\square_{57} (4(\partial_{14}\partial_{38})) + \square_{14}\square_{57} (-4(\partial_{23}\partial_{28}) + 5(\partial_{38}\partial_{56})) \\
 & + \square_{15}\square_{38} (6(\partial_{46}\partial_{78})) + \square_{16}\square_{38} (6(\partial_{34}\partial_{57})) + \square_{16}\square_{34} (-4(\partial_{27}\partial_{28})) \\
 & + \square_{34}\square_{56}\square_{57}(2) + \square_{38}\square_{56}\square_{78}(-3) + \square_{34}\square_{38}\square_{56}(-2) \\
 & + \square_{34}\square_{56}\square_{78}(5/2) + \square_{34}\square_{38}\square_{46}(1) + \square_{16}\square_{38}\square_{78}(3/2) \\
 & + \square_{16}\square_{34}\square_{78}(5/2) + \square_{14}\square_{56}\square_{78}(3) + \square_{16}\square_{34}\square_{57}(1) \\
 & + \square_{14}\square_{38}\square_{57}(17/2) + \square_{14}\square_{38}\square_{56}(3)
 \end{aligned} \tag{28}$$

It is, of course, easy to sort the six-loop and the four-loop parts of our list of derived topologies into numerators for the masters 1,2 and the five loop cases.

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